CSE311 Microwave Engineering

LEC (11)

Rectangular Waveguides

Example 5.1

An air-filed copper rectangular waveguide has dimensions a = 2 cm and b = 1 cm. Determine the range of frequencies over which the waveguide will operate single mode (TE_{10}) mode.

Solution:

From (5.42) the frequency for TE is given by: $f_{c_{mn}} = \frac{C_E}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

- Since the waveguide is air filled then for mode , TE_{10}, n =1 and m=0, we get:

$$f_{c_{10}} = \frac{C}{2a} = \frac{3x10^8}{2x2x10^{-2}} = 7.5 \ \ GHz$$

- The next higher-order mode will either TE_{20} or TE_{01}. From (5.42) we have the same cutoff frequency for both modes, since a = 2b which is given by:

$$f_{c_{20}} = \frac{C}{2\pi}\sqrt{\left(\frac{2\pi}{a}\right)^2} = \frac{C}{a} = f_{c_{01}} = \frac{C}{2\pi}\sqrt{\left(\frac{\pi}{b}\right)^2} = \frac{C}{2b} = 15 \ \ GHz$$

Thus the operating range of frequencies over which the waveguide will operate single mode is 7.5 GHz < f < 15 GHz.

Example 5.2

An air-filed copper rectangular waveguide has dimensions a = 4.5 cm and b = 9 cm. Determine:

a) The cutoff wavelength λ_c for the dominant mode.

b) The phase velocity V_p in the waveguide at 1.6 times the cutoff frequency.

c) Repeat (a) and (b) if guide filled dielectric having (μ_r= 1 and ε_r = 1.7).

Solution:

From (5.45), the wavelength λ_c for TE is given by:

$$\lambda_{c_{mn}} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

a) The dominant mode is TE_{10} or TE_{01}.

For TE_{10} m = 1 and n = 0, so λ_c is: λ_c = 2a = 9 cm.

For TE_{01} m = 0 and n = 1, so λ_c is: λ_c = 2b = 18 cm.

Therefore, the cutoff wavelength λ_c = 2b = 18 cm.

b) From (5.44), V_p at 1.6 times the cutoff frequency is given by:

$$v_p = \frac{C_{Dielectric}}{\sqrt{1-\left(f_{c_{mn}}/f\right)^2}} = \frac{C}{\sqrt{1-(1/1.6)^2}} = 1.28\,c \quad m/s$$

c) λ_c for (μ_r= 1 and ε_r = 1.7) is given by : $\lambda_{c_{01}} = 2x9\sqrt{\varepsilon_r} = 23.5 \ cm$

Vp for (μr= 1 and εr = 1.7) is given by : $v_p = \frac{C/\sqrt{1.7}}{\sqrt{1-(1/1.6)^2}} = 0.98\,c \quad m/s$

Example 5.3

Consider a copper K-band rectangular waveguide Teflon-filed having (μ_r= 1 and ε_r = 2.08), if its dimensions are: a = 1.07 cm and b = 0.43 cm do the following:

a) Calculate The cutoff frequency, f_c, for the first five modes (TE_{10}, TE_{20}, TE_{01}, TE_{11} and TE_{21}) to propagate.

b) At f = 15 GHz, determine the propagating mode.

c) Calculate: the wave number, k, the cutoff wave number, k_c, the propagation constant, β, the cutoff wavelength, λ_c, waveguide wavelength, λ and the phase velocity, V_p.

Solution:

a) From (5.42) the frequency for TE is given by:

$$f_{c_{mn}} = \frac{C_E}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

These values are listed in the table shown.

Mode	m	n	F_c (GHz)
TE_{10}	1	0	9.72
TE_{20}	2	0	19.44
TE_{01}	0	1	24.19
TE_{11}	1	1	26.07
TE_{21}	2	1	31.03

b) At f = 15 GHz, from the listed values in the table it is clear that $f_{c10} < f < f_{c20}$. Therefore, Mode TE_{10} is the propagating mode in this waveguide

Example 5.3 Solution:

c) the wave number, k, is:

$$k = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu\varepsilon} = 2\pi x15x10^9\sqrt{4\pi x10^{-7}x\frac{1}{36\pi}x10^{-9}x2.08} = 453.1\ \ m^{-1}$$

The cutoff wave number, k_c is:

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \sqrt{\left(\frac{\pi}{a}\right)^2 + 0} = \frac{\pi}{1.07x10^{-2}} = 293.6\ \ m^{-1}$$

The propagation constant, β is:

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{\omega^2\mu\varepsilon - k_c^2} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \sqrt{(453.1)^2 - (293.6)^2} = 345.1\ \ m^{-1}$$

The cutoff wavelength, λ_c, is: $$\lambda_{c_{10}} = \frac{2\pi}{k_c} = \frac{2\pi}{293.6} = 2.14\ \ cm$$

The waveguide wavelength, λ is: $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{345.1} = 1.82\ \ cm$$

The phase velocity, V_p is: $$v_p = \frac{\omega}{\beta} = \frac{2\pi x15x10^9}{345.1} = 2.73x10^8\ \ m/s$$

Example 5.6

Consider a copper K-band rectangular waveguide Teflon-filed having (μ_r= 1 and ε_r = 2.08), if its dimensions are: a = 1.07 cm and b = 0.43 cm do the following:

a) Calculate The cutoff frequency, f_c, for the first five modes (TE_{10}, TE_{20}, TE_{01}, TM_{11} and TM_{21}) to propagate.

b) At f = 30 GHz, determine the propagating modes.

c) At f = 15 GHz, calculate: the wave number, k, the cutoff wave number, k_c, the propagation constant, β, the cutoff wavelength, λ_c, waveguide wavelength, λ and the phase velocity, V_p.

Solution:

a) From (5.42) and (5.71) the frequency for TE or TM mode is given by:

$$f_{c_{mn}} = \frac{C_E}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

These values are listed in the table shown.

Mode	m	n	F_c (GHz)
TE_{10}	1	0	9.72
TE_{20}	2	0	19.44
TE_{01}	0	1	24.19
TE_{11}, TM_{11}	1	1	26.07
TE_{21}, TM_{21}	2	1	31.03

b) At f = 30 GHz, from the listed values in the table it is clear that TE_{10}, TE_{20}, TE_{01}, TE_{11}, TM_{11} are the propagating modes in this waveguide

Example 5.6 Solution: c) the wave number, k, is:

$$k = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu\varepsilon} = 2\pi x 15x10^9\sqrt{4\pi x10^{-7}x\frac{1}{36\pi}x10^{-9}x2.08} = 453.1\ m^{-1}$$

The cutoff wave number, k_c is:

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \sqrt{\left(\frac{\pi}{a}\right)^2 + 0} = \frac{\pi}{1.07x10^{-2}} = 293.6\ m^{-1}$$

The propagation constant, β is:

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{\omega^2\mu\varepsilon - k_c^2} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} = \sqrt{(453.1)^2 - (293.6)^2} = 345.1\ m^{-1}$$

The cutoff wavelength, λ_c, is: $$\lambda_{c_{10}} = \frac{2\pi}{k_c} = \frac{2\pi}{293.6} = 2.14\ cm$$

The waveguide wavelength, λ is: $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{345.1} = 1.82\ cm$$

The phase velocity, V_p is: $$v_p = \frac{\omega}{\beta} = \frac{2\pi x15x10^9}{345.1} = 2.73x10^8\ m/s$$